by Charles H. Gushee

How to Hedge A Bond Investment

Suppose that, when you buy a bond, you want to keep fully invested at a known yield. If your bond is a zero coupon bond that matures at the end of the period for which you want to keep fully invested, you can lock in the bond's initial yield simply by holding it to maturity. If your bond has a positive coupon rate, however, you must reinvest the coupon each six months as you receive it. But if you don't know what the market yield will be at those future reinvestment dates, how can you know exactly what your bond fund will amount to?

A five-year coupon bond may be thought of as a portfolio of 11 zero coupon bonds—10 pieces being the semiannual coupon payments and the eleventh being the principal payment at maturity. The average of the maturities on these 11 zero coupon bonds, weighted by their present worths, is called the duration. With a zero coupon bond, the maturity of the bond equals its duration. On more complicated bonds, duration is less than maturity. The initial yield can still be locked in, but for a period equal to the bond's duration, rather than its term to maturity. This creates certain complications in practice.

For example, a five-year, seven per cent bond priced to yield eight per cent has a duration of 4.3 years. If the investor's bond portfolio is to behave like a zero coupon bond with a maturity of 4.3 years, then six months hence its duration must be one-half year less, or 3.787 years. A seven per cent, 4½-year bond yielding eight per cent has a duration, however, of 3.924 years. Obviously, adjustments must be made from time to time to maintain the desired duration. The author provides examples of how these adjustments can lock in initial yields.

IF AN INVESTOR buys a bond and wishes to keep his funds fully invested, he must reinvest the coupon interest each six months, as it is received. But he won't know what the market yield rate will be at those future reinvestment dates. So how can he know exactly what his bond fund will amount to at some future date, or what his overall yield will be? Up to now, he couldn't. But a new technique allows an investor to know for sure on the day of his first investment what his yield will be and the sum he will have at any future target date. He will have immunized his bond investment against future changes in market yield levels.

This immunization is accomplished by use of a measure we will call "duration," first suggested only 40 years ago by Frederick Macaulay.[1] Duration is the time for which an investment can be considered to be fully invested (that is, including if necessary the automatic reinvestment of any interest).

The Concept of Duration

Consider the popular Certificate of Deposit, which has a guaranteed rate for a guaranteed

1. Footnotes appear at end of article.

Charles Gushee is past President and current Chairman of the Board of the Financial Publishing Company, Boston.

Table I Computation of Duration (Seven Per Cent, Five-Year Bond Yielding Eight Per Cent)

Term in Years	*Present Worth of \$1.00 at 8%*	*Value of Each Piece at Maturity*	*Present Worth of Each Piece*	*Multiply by Weighting*	*Weighted Present Worth*
½	\$0.961538	35	\$33.65	½	\$16.83
1	0.924556	35	32.36	1	32.36
1½	0.888996	35	31.11	1½	42.66
2	0.854804	35	29.92	2	59.84
2½	0.821927	35	28.77	2½	71.92
3	0.790315	35	27.66	3	82.98
3½	0.759918	35	26.60	3½	93.10
4	0.730690	35	25.57	4	102.28
4½	0.702587	35	24.59	4½	110.65
5	0.675564	35	23.64	5	118.20
5		1,000	675.56	5	3,377.80
			\$959.43		\$4,112.62

$$\frac{\$4{,}112.62}{\$959.43} = 4.2865 \text{ Years Duration}$$

term. The investment grows at that rate, with no payouts until maturity. The guaranteed term is the CD's duration. United States Savings Bonds also illustrate pure growth from purchase price to maturity price, with the term of the Savings Bond being its duration.

Suppose an investor could buy a five-year, zero per cent coupon bond at the market yield of seven per cent, paying \$708.92 for \$1,000 maturity or face value. There is no way he can avoid earning seven per cent for five years, no matter what the market does. (This, of course, ignores commissions.) If, by some chance, he had to switch investments along the way, he could sell at the market yield rate and buy another zero coupon piece of paper at the same yield rate and price.

To carry the thinking one step further, suppose that, at the original investment date, our investor cannot find a five-year, zero per cent bond, but can invest equal amounts in three-year and seven-year zero per cent bonds. They average to the five-year term he wants. This also constitutes an immunized investment, but only provided the investor can switch sometime along the way into zero per cent bonds with the desired maturity of five years from the original investment date. The switch can be made at any market rate whatsoever, but it must be made at the same rate from a short (three-year) to a longer term and from a long (seven-year) to a shorter term.

Of course, zero per cent coupon bonds are very rare, if they exist at all. But if our investor bought a five-year coupon bond, he could, figuratively, cut the bond into 11 pieces of paper —10 pieces being the coupons and the eleventh the face. He has thus created 11 different zero per cent bonds. Just as he averaged three-year and seven-year zero per cent bonds, he can average the 11 pieces; this time, however, he must create a weighted average, because the pieces are different, both in term and in present worth. The resulting weighted average is the duration of the bond.

The mechanical computation consists of finding the present worth of each of the (in this case) 11 pieces of paper. The sum (unweighted) is the ordinary bond value. However, weighting by multiplying each present worth by its term in years and then dividing the weighted total by the ordinary bond value yields the bond's duration. Table I shows the computation for a five-year, seven per cent bond priced to yield eight per cent.

Duration vs. Term

Instead of thinking of this bond as a five-year term bond, we can think of it as the equivalent of a fully invested fund with a duration of 4.3 years. In other words, the bond can be thought of as an immunized investment, but its eight per cent yield is guaranteed only for 4.3 years, not for the full five years. For the guarantee to work, the fund must always be kept invested for the remaining duration; that is, one year later, the duration must be 3.3 years. This requires switching at any yield rate (market up or market down), but always at the same yield rate in and out. A table of duration is an essential guide for this purpose.

Table II shows the duration schedule for six per cent bonds. A few conclusions are obvious. First, duration is always shorter than the term of

Table II Duration Schedule for Six Per Cent Coupon Bonds

Yield	1 Year	2 Years	5 Years	10 Years	Term 15 Years	20 Years	30 Years	40 Years	Limit
2%	0.986	1.918	4.453	8.045	11.108	13.810	18.472	22.436	50.500
3%	0.986	1.917	4.438	7.953	10.865	13.349	17.409	20.605	33.833
4%	0.986	1.916	4.423	7.859	10.615	12.876	16.340	18.822	25.500
5%	0.986	1.915	4.408	7.762	10.358	12.393	15.283	17.128	20.500
6%	0.985	1.914	4.393	7.662	10.094	11.904	14.253	15.553	17.167
7%	0.985	1.913	4.377	7.559	9.825	11.413	13.263	14.118	14.786
8%	0.985	1.912	4.361	7.454	9.552	10.922	12.324	12.829	13.000

Table III Duration Schedule for Two Per Cent Coupon Bonds

Yield	1 Year	2 Years	5 Years	10 Years	Term 15 Years	20 Years	30 Years	40 Years	Limit
2%	0.995	1.970	4.783	9.113	13.033	16.582	22.702	27.719	50.500
3%	0.995	1.970	4.777	9.061	12.870	16.223	21.687	25.721	33.833
4%	0.995	1.970	4.770	9.007	12.695	15.837	20.595	23.628	25.500
5%	0.995	1.969	4.763	8.950	12.508	15.422	19.941	21.505*	20.500
6%	0.995	1.969	4.756	8.891	12.310	14.981	18.244*	19.421*	17.167
7%	0.995	1.969	4.749	8.828	12.100	14.515	17.026*	17.440*	14.786
8%	0.995	1.968	4.742	8.762	11.879	14.026*	15.811*	15.610*	13.000

*Durations greater than limit.

a bond (except at six months, when the two are equal). Duration has to be shorter because the coupons start maturing in six months. Second, the longer the term, the greater the difference between term and duration. This is because, whereas the term can increase indefinitely, duration always approaches a limit. This limit is the reciprocal of the yield rate plus one-half year. Thus, for a bond to yield eight per cent, the limit of duration is 13 years (1/0.08 + 1/2). Obviously, the limit decreases as yield increases.

The limit is independent of the coupon rate. However, when the coupon rate is the same as or greater than the yield rate (i.e., price 100 or greater), duration approaches the limit directly. Conversely, for discount priced bonds, duration can increase beyond the limit and then recede to the limit. Table III, showing the duration schedule for two per cent coupon bonds, illustrates this.[2]

Immunizing a Bond Portfolio

To repeat, immunization requires keeping the fund invested so that its duration reaches the original target date. In the case of a seven per cent, five-year bond yielding eight per cent, duration equals 4.287 years. That is, we could immunize to a target date 4.287 years from the date of original investment. But six months later, we want a duration one-half year less, or 3.787 years. Reference to the table shows that a seven per cent, 4½-year bond yielding eight per cent has a duration of 3.924 years. Somehow we must get the duration down from 3.924 to 3.787 years.

Sometimes the investment of the first matured coupon in a very short bond will reduce the duration sufficiently. Sometimes switching a small amount of the original bonds into shorter bonds will do it. The extreme, of course, would be to sell all the 3.924-year duration bonds and buy new bonds with a duration of 3.787 years.

Of course, the above solutions assume the original yield rate, market level unchanged. But whatever the market level, immunization can be maintained by determining the duration of the fund at that market level and adjusting the fund to the desired duration by switches at that same market yield level. It matters not a whit that the market has gone up or down. No matter what happens, the original yield has been immunized for the original duration.

Selling All Bonds

Admittedly, it is a nuisance to keep immunized with bonds. It means that some action, if only reinvestment of coupon income, must be taken every six months. The scheme will work best on large portfolios, where the switch of a few bonds can bring the portfolio to the desired duration and where a computer program can do the testing. But the simplest case for purposes of illustration is that in which a single-issue

Table IV Immunization By Sale of Portfolio
How to Reach a Target of $100,000 Five Years Hence by Buying and Selling Portfolios of Seven Per Cent Bonds

Time To Target (Duration)	*Market Yield Level*	*Sell*	*Buy*	*Present Worth at Market*
5 Years	6.00%	—	With initial fund of $74,409 Buy 7%s to yield 6.00% Term 5.96 years	$74,409
4½ Years	5.90%	Sell 7%s to yield 5.90% Collect coupon Proceeds $76,977	Buy 7%s to yield 5.90% Term 5.28 years	$76,977
4 Years	5.80%	Sell 7%s to yield 5.80% Collect coupon Proceeds $79,559	Buy 7%s to yield 5.80% Term 4.56 years	$79,559
3½ Years	5.70%	Sell 7%s to yield 5.70% Collect coupon Proceeds $82,144	Buy 7%s to yield 5.70% Term 3.91 years	$82,144
3 Years	5.60%	Sell 7%s to yield 5.60% Collect coupon Proceeds $84,732	Buy 7%s to yield 5.60% Term 3.29 years	$84,731
2½ Years	5.50%	Sell 7%s to yield 5.50% Collect coupon Proceeds $87,317	Buy 7%s to yield 5.50% Term 2.69 years	$87,315
2 Years	5.40%	Sell 7%s to yield 5.40% Collect coupon Proceeds $89,894	Buy 7%s to yield 5.40% Term 2.11 years	$89,891
1½ Years	5.30%	Sell 7%s to yield 5.30% Collect coupon Proceeds $92,457	Buy 7%s to yield 5.30% Term 1.55 years	$92,453
1 Year	5.20%	Sell 7%s to yield 5.20% Collect coupon Proceeds $94,999	Buy 7%s to yield 5.20% Term 1.02 years	$94,996
½ Year	5.10%	Sell 7%s to yield 5.10% Collect coupon Proceeds $97,517	Buy 7%s to yield 5.10% Term ½ year (Face amount $96,622)	$97,513
Target		Collect face $96,622 Collect coupon 3,382 $100,004		$100,000

portfolio is sold every six months and the proceeds reinvested in a single issue with a term a few months shorter (a bond salesman's dream).

Our example will use seven per cent bonds exclusively, assuming there are lots of them available in the market. Our investor starts with a target of $100,000 five years hence, and with a six per cent market yield level. His initial investment is thus $74,409—the present worth of $100,000 payable five years hence at six per cent compounded semiannually. We'll assume he can invest in any face amount of bonds, not limited to even $1,000 apiece. (With a big portfolio, an average residue left at interest in a bank will not cause a significant deviation.) Furthermore, we'll arbitrarily assume that yields decline 0.10 per cent each six months.

Remember that the use of duration is the key to the solution. Since seven per cent, six-year bonds yielding six per cent have a duration of 5.029 years, and our investor wants a five-year duration, interpolation produces a term of 5.96 years. Our investor therefore uses his $74,409 to buy $70,902 face amount of seven per cent, 5.96-year bonds yielding six per cent, at a unit price of 104.946.

Six months later, the market yield level has moved from six to 5.90 per cent. The investor sells out, receiving $76,991, plus coupon income of $2,568, so that the fund totals $76,977. With

Table V Immunization by Switching
How to Reach a Target of $100,000 Five Years Hence by Switching from Long to Short Seven Per Cent Bonds

Time to Target (Duration)	*Market Yield Level*	*Value of Long Bonds (A)*	*Term of Long Bonds*	*Value of Short Bonds (Z)*	*Term of Short Bonds*	*Coupon Income*	*Total in Fund*	*Present Worth of $100,000*	*Value of Longs Switched to Shorts*
5 Years	6.00%	$74,409	5.96 yrs.	—	5.0 yrs.	—	$74,409	$74,409	—
4½ Years	6.10%	$73,825	5.46 yrs.	—	4.5 yrs.	$2,482	$76,307	$76,308	$12,106
4 Years	6.20%	$61,268	4.96 yrs.	$14,588	4.0 yrs.	$2,567	$78,423	$78,330	$16,097
3½ Years	6.30%	$44,872	4.46 yrs.	$32,990	3.5 yrs.	$2,658	$80,520	$80,485	$10,527
3 Years	6.40%	$34,138	3.96 yrs.	$45,925	3.0 yrs.	$2,752	$82,815	$82,779	$8,979
2½ Years	6.50%	$25,029	3.46 yrs.	$57,385	2.5 yrs.	$2,848	$85,262	$85,222	$7,803
2 Years	6.60%	$17,143	2.96 yrs.	$67,768	2.0 yrs.	$2,948	$87,859	$87,821	$6,776
1½ Years	6.70%	$10,326	2.46 yrs.	$77,248	1.5 yrs.	$3,051	$90,625	$90,587	$5,118
1 Year	6.80%	$5,191	1.96 yrs.	$85,220	1.0 yrs.	$3,158	$93,569	$93,531	$3,398
½ Year	6.90%	$1,788	1.46 yrs.	$91,645	0.5 yrs.	$3,269	$96,702	$96,665	$1,788
Target				Collect face	$96,655				
				Collect coupon	3,383				
					$100,038				

the market yield level now at 5.90 per cent, the present worth of $100,000 due in 4½ years is $76,977. So he is at the exactly proper point.

Now our investor finds that a duration of 4½ years for seven per cent bonds on a 5.90 per cent yield basis represents a term of 5.28 years, so he invests his fund in seven per cent bonds yielding 5.90 per cent for a term of 5.28 years. And so, on and on, each six months he sells out at the market and invests at the same market yield in bonds with terms a few months shorter than the bonds he sold. Table IV shows the sell-out proceeds and the coupon income: The investor will come out with his target $100,000 plus four dollars to spare. Although the market yield has fallen from six to 5.10 per cent over the five-year investment period, he has immunized his initial investment at six per cent.

Immunization by Switching

In the previous example, our investor sold the entire bond portfolio each six months at the prevailing market yield level and bought new bonds of slightly shorter term but of the desired duration. It is simpler to adjust the bond portfolio by switching a few long bonds into shorter bonds to obtain the desired duration. This does involve a new calculation by weighted average, but it is required only once in six months, and the arithmetic, although a nuisance, is elementary.

Suppose our investor starts with the same issue as before — seven per cent bonds, six per cent market yield, a 5.96-year term representing five years duration to target date, and initial funds of $74,409. The previous example showed yields declining each six months, so this time we'll assume change in the opposite direction. And we'll suppose the investor can switch to one other issue, so that each six months he sells a few bonds of the original (longer) term and invests the proceeds along with the coupon income in the shorter issue.

Table V shows the immunization procedure over the five years, omitting such nonessentials as unit basis price and face amounts, which can be inferred from the figures shown. The table shows the value of the fund at the prevailing yield level each six months, the value for each of the two bonds held and the coupons collected. If the total fund equals the present worth of $100,000 for the time to target, at the prevailing market yield level, it is immunized; the table shows that it does, approximately.

The right-hand column of the table shows the action our investor must take to maintain immunization. He must always switch from the longer issue in the value shown, investing this amount, together with the coupons, in as much of the shorter issue as it will buy. Thus, when each six-month date rolls around, he identifies the market yield level and values the two bonds at this yield rate, and with the coupons; the total is proved out to the present worth. Then he switches a few bonds from longs to shorts, as indicated in the right-hand column of the schedule. (We have again assumed that bonds are not restricted to $1,000 pieces, but are available in any denomination.)

Determining the necessary switch is simple algebra; the formula can be produced from the construction of a weighted average of duration. Let A equal the value of the long (original) bond, Z the value of the short (ending) bond, C the coupon income for one-half year, D the duration (identified by subscript as D_A, D_Z, D_n, where n is time to target date) and x the value, not face amount, of bonds to be switched. Then the necessary switch will be:

$$x = \frac{A(D_A - D_n) + (Z+C)(D_Z - D_n)}{D_A - D_Z} .$$

At 4½ years, for example, A has a term of 5.46 years and a duration of 4.63 years and Z has a term of 4.50 years and a duration of 3.95 years. So the value of bonds to be switched is:

$$x = \frac{73{,}825(4.63-4.50) + (0+2{,}482)(3.95-4.50)}{4.63-3.95} ,$$

$$x = \frac{9{,}597 - 1{,}365}{0.68} = \$12{,}106.$$

The essential thing is always to keep the fund invested at an average term that produces a duration equal to the time to target date.[3] Any technique that accomplishes this can be utilized. In our first example, the investor sold out the entire fund every six months and bought a slightly shorter bond. In the second example, he utilized two bond issues, switching a few bonds from the longer to the shorter issue every six months. Sometimes, reinvesting the coupons from a long bond in a shorter bond will reduce duration to the proper extent; solving the weighted average formula to obtain the necessary term is easy, but the result may be less than zero, hence useless.

Caveats

Commissions and taxes are always with us. To the extent that immunizing forces switches, it may increase commissions, but not by much. In our example, an initial $74,409 investment was completely switched over the course of five years. Assuming one per cent cost, in and out, for the whole investment, plus 0.50 per cent for investing the coupons, the whole expense might run to $870, or perhaps 0.20 per cent average per year, which is not excessive.

Insofar as taxes are concerned, yields, immunized or not, should be known after taxes. For a tax-exempt investor, immunizing will not change anything. For an investor in tax-exempt bonds, the income from interest will be exempt. There may be tax on capital gains from necessary switching, but it will be at lower rates and can be offset by capital losses (discount bonds, entire loss or premium bonds net of mandatory amortization). A corporate investor will pay the same rates, immunized or not, and the procedures for immunization will not change things significantly.

The Yield Curve

Bonds of different maturities will normally have different yields. Yields are usually lowest for short maturities, with yields gradually rising, then flattening out, for longer maturities. The rationale for a rising yield curve is that risk increases, the longer the time to repayment. However, the shape of a bond yield curve fluctuates widely under varying market conditions. How does this affect immunization?

At first it would seem that there should be no effect. After all, the whole purpose of immunization is to insure against changes in yield levels. If anything, one might expect that the nature of the yield curve would allow an increase in return. For a 10-year target date, an investor might use 17-year bonds having a 10-year duration; and the 17-year bonds, being longer, might carry a greater yield than the 10-year bonds. Thus immunization would automatically mean a higher rate.

The problem with the yield curve, of course, is that bonds must constantly be switched from a longer to a shorter maturity at the same yield rate. These switches buck the trend of a (normally) rising yield curve. Fortunately, it is typically only a very small buck. In our first example, an investor sold and bought new the whole portfolio every six months, but the largest decline in term was less than three months, and this isn't much on the yield curve. In the second example, our investor switched between two bond issues that were less than one year apart. A year, also, isn't too much on the yield curve. At very short terms, where the curve is steepest, the switch is smallest, and the market value will change least for a given change in yield. Thus the yield curve is not a barrier to immunization — only a slight impediment.

Indeed, aberrations in the yield curve can even produce profit above the immunized return. Recently, for example, tight money has forced short yields to extremely high levels. An investor immunizing on an eight per cent yield basis might well initially invest in short, six-month paper yielding 12 per cent. The very short duration might serve to bring average duration down to the needed point, or close to it, and the higher yield would produce extra income for the fund.

One last word of caution, however: Most bonds issued in recent years carry high, mongrel rates (part interest rate, part inflation rate) and protective call provisions. This makes them unsuitable for immunization. A bond callable at any coupon date has, for practical purposes, a duration of one-half year. However, bonds at the long end of serial issues, such as equipment trust bonds, are rarely callable. Also, older deep discount bonds are everywhere, and will probably not be called or, if called, will profit the fund. Some searching will usually reveal suitable bonds for purposes of immunization at normal yield levels. ■

Appendix

Development of Duration

Within a few years of the publication of Macaulay's book in 1938, other economists independently re-invented the concept of duration. In 1939, the British economist Sir John R. Hicks published *Value and Capital*, in which he discussed the effects of changes in the interest rate on the economy.[4] For this purpose he invented a measure for what he called "elasticity," naming it "average maturity." He defined average maturity as:

$$\frac{BX_1 + 2B^2X_2 + 3B^3X_3 + \ldots\ldots}{X_0 + BX_1 + B^2X_2 + B^3X_3 + \ldots\ldots},$$

where B equals $1/(1+i)$ and the Xs are future payments.

This is the same as Macaulay's duration, except for the X_0, or instant payment, in the denominator. If used for bonds (and Hicks does so use it), this formula would consider a bond on coupon date "coupon on." As bond values are customarily "coupon off," Macaulay's formula is the better implement for bonds. The limiting duration according to Hicks would be $1/i$, versus $1/i+1/2$ year for Macaulay.

Hicks wanted to know what happens to an economy when the level of interest rates changes. He decided that the duration of any asset constitutes the measure of the volatility in present worth caused by interest rate fluctuations. The duration of any investment can therefore define its volatility.

Similarly, Paul Samuelson, in the March 1945 issue of *American Economic Review*, originated a theory of duration in his discussion of "The Effect of Interest Rate Increases on the Banking System." Samuelson's duration was termed "weighted average time period of payment." He found that "increased interest rates will help any organization whose (weighted) average time period of disbursements is greater than the time period of its receipts." In our example of immunization, duration of disbursements and receipts are equal; in other words, duration of investments is made to equal a target date disbursement.

Actuarial Approach

A third, and again entirely independent, development of duration came from insurance actuaries. Life insurance companies use all sorts of "matching" techniques, whereby they attempt to offset a liability at a given future date with invested assets. The first general analysis of immunization, by F.M. Redington, appeared in 1952.[5] It is simple, elegant and of such general application that its algebraic presentation is more comprehensible than a verbal explanation.

The present worth at rate i of assets (or income) is denoted by V_A, and the present worth of liabilities as V_L. Obviously, $V_A - V_L$ is wealth, but for our purposes we discard excess assets and assume that, at rate i, V_A equals V_L. If the interest rate changes by h, we want to assure that $V_{A,i\text{-}h}$ equals $V_{L,i\text{-}h}$; the life insurance company is thereby immunized against a change of rate. It can be proved mathematically that this is true for a change of h in either direction, if durations of assets and liabilities are equal, but always provided the spread of assets is greater than the spread of liabilities (i.e., we can't have a liability coming due before an offsetting asset does).

This suggests a whole theory of speculation in fixed payment securities. To immunize, an investor should keep the duration of borrowed funds the same as the duration of invested funds. If he thinks interest rates are going up, and values down, however, he could keep the duration of his borrowed funds longer than the duration of his invested funds. Conversely, if he thinks interest rates are going to decline, and values go up, he will win if the duration of his borrowings (or liabilities or disbursements) is shorter than the duration of his investments (or assets or income). In either case, however, he will have abandoned immunization in favor of gambling on market trends.

Footnotes

1. Frederick Macaulay, *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the United States Since 1856* (New York: National Bureau of Economic Research, 1938).

2. Tables of duration were not generally available until recently. Tables by Alfred Weinberger and Henry Nothof of Sun Life Assurance Company of Canada are now available from the Financial Publishing Company (82 Brookline Avenue, Boston 02215) for $50 (for Publications No. 761, 751 and supplement). The supplement includes tables for coupons up to 25 per cent and yields up to 28 per cent. A 25 per cent bond priced to yield 25 per cent and due in 10 years has a duration of 4.07 years. If the 25 per cent coupon runs for 29 years, the duration will have reached its maximum at 4.50 years.
3. A weighted average can be constructed at any point to check that the fund has the desired duration and the correct switch. At the end of two years, for example, long bonds will be 25,159 (34,138 − 8,979) and short bonds 57,656 (45,925 + 2,752 + 8,979). The weighted average will be:

$$\begin{array}{ll} 25{,}159\ (3.55) = & 89{,}314 \\ \underline{57{,}656\ (2.76) =} & \underline{159{,}130} \\ 82{,}815 & 248{,}445 \end{array}$$

$$248{,}445/82{,}815 = 3\ \text{Years},$$

which is the correct duration.
4. J.R. Hicks, *Value and Capital* (Oxford: Clarendon Press, 1939).
5. Redington, "Review of the Principles of Life-Office Valuations," *Journal of the Institute of Actuaries* (London), Vol. 78, No. 3.

Letters continued

risk and return. I am satisfied that the conclusions reached using these assumptions remain correct.

Mr. Stromquist is also helpful in pointing out that some of the article's results will depend upon the assumed rate of return on the stock. I have inspected the results obtained when the stock return exceeds the risk-free rate and most but not all of the conclusions remain unchanged. A careful accounting of this rate of return effect might be useful. It should be noted, however, that while in the last 10 years some stocks have yielded a return greater than the risk-free rate, many others have yielded the same or a lesser return. Making a neutral assumption of a risk-free return on the stock over the assumed six-month investment period for these option positions, while not necessarily the only possible assumption, is not an unreasonable one. The article's conclusions, then, may continue to be viewed as correct for stocks having an expected return over the next six months about equal to the risk-free rate (as assumed at the outset). In today's interest rate environment, such a performance would be exceptional and welcomed.

Finally, I question whether by "simplifying assumptions" investors in general will be made indifferent between strategies with fairly priced options, as Mr. Stromquist claims. Most portfolio managers, myself included, find considerable differences between option strategies whether options are fairly priced or not. These differences relate to factors other than risk and return. This very fact points out a major limitation of this and so many other analyses of risk and return, namely the artificial limitation of investment decisions to these two variables. It seems that a knowledge of risk and return, while providing a useful initial point for analyzing investment decisions, is more often than not, incomplete until additional factors and preferences are included. A small attempt at broadening the number of decision variables was made purposely in the article when the full return profiles of the optioned positions were sketched. These return profiles contain information over and above the risk and return quoted in tables.

–Ronald T. Slivka
Vice President
Morgan Guaranty Trust Company
New York

The Care and Feeding of Buy-Side Analysts

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ing progress with them, the check arrives. The euphoria disappears as they slink back to their lairs.

So, for lunch, you want an analyst who realizes that 500 million Chinese don't care whether GM is $102 or $62, who knows that a day in his life is 24 hours long—that it does not open at 10 A.M. and close at 4:00 P.M. with a two-hour hiatus at noon. An analyst who is able to understand that some of the business principles determining Capital Cities' results are evident in Time's numbers as well, and who can laugh. Preferably when his mouth isn't full.

My best advice for dinners is to make up a diplomatic excuse for skipping the whole thing entirely. After all, it takes an hour and a half to get home, during which time you have to read some of the 40 publications to which you subscribe, so who needs another two hours' work? And dinner with a client is work, even as the sun sets and the wine flows and the inhibitions ebb. You still have to turn up at the office the next morning, grind out research reports that go mostly ignored and telephone calls about events the world may little note nor long remember.

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When the requests for contributions to Wharton come in the mail, I ponder all the practical tools with which the School of Finance and Commerce equipped me. I ponder, but come up empty. They never taught me the care and feeding of buy-side analysts. ■